Karanjia Auto College,Karanjia,Mayurbhanj

CC 5

Mathematical Physics – II

I. One mark questions.

1. The period of cos x is _____ .
2. The Quotient of any two odd function is _____ .
3. For a periodic function $f(x + T) = f(x)$. (T/F)
4. The product of any two even function is even. (T/F)
5. The product of any even and any odd function is odd. (T/F) 6. If $f(x)$ is defined on [-L, L] then represent Fourier series of $f(x)$.
7. Is $f(x)= x^2$ an even function ?
8. $J_n(x) = (-1)^n J_n(x)$ (T/F)
9. In the equation $x^3 y'' + 3xy' + 6y = 0$, what is the singular point ? Is it regular or irregular.
10. Write Rodrigues formula for Legendre polynomial.
11. Write the value of $p'_{n+1}(x) - p'_{n-1}(x)$.
12. State the condition of orthogonal property of Legendre polynomials.
13. Write the expression for the generating function of Hermite polynomials.
14. Write Laguerre differential equation.
15. $(x^2-1)P'_n(x) = n[\_\_\_\_]$.
16. $P'_{n-1}(x) = xP'_n(x) - _____$.
17. $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$. Write the correct relation ?
18. What is the value of $P_3'(x)$?
19. Write Associated Legendre's differential equation.
20. Write the expression for associated legender polynomials.
21. Write Rodrigues formula for associated legendre polynomials.
22. The value of beta function is given by $\beta(m, n) = _____$.
23. The error function is represented by $erf(x) = _____$.
24. Is beta function symmetric ?
25. State the relation between beta and gamma function.
26. What is the value of $\beta(9, 15) - \beta(15, 9)$.
27. Write the expression for the dipole moment acquired by the conducting sphere placed in an external uniform electric field ?
28. Write the expression for the fundamental frequency of vibration in a stretched string ?
29. A dielectric sphere is placed in an uniform electric field. Then which is greater, the electric field inside the sphere or outside the sphere ?
30. If electric potential V is a function of distance (r) only, then write Laplace equation in cylindrical coordinates.

31. Write the expression for dipole moment due to a dielectric sphere placed in a uniform electric field for outside point.

II. 1.5 mark questions.

1. Find the period of $\sin \pi x/l$.
2. Show that $f(x) = (1 + x)$ is neither odd nor even.
3. Find the value of a_0 and a_n for $f(x) = x^2, -2 \leq x \leq 2$.
4. If $f(x)$ and $g(x)$ have period p, then what is the period of $af(x) + bg(x)$ where a and b are constants.
5. Write the expression for Fourier expansion of $f(x)$ in case $f(x)$ is even. What is the expression if $f(x)$ is odd.
6. Write the even extension of the function defined by $f(x) = x(\pi - x)$ for $0 \leq x \leq \pi$.
7. Define ordinary point and singular point.
8. What do you mean by regular singular point ?
9. Write the normal form of homogeneous linear second order differential equation.
10. What is the type of singular point in the equation $xy'' + y + xy = 0$.
11. Write the importance of Frobenius method of power series solution.
12. Form the expression of $H_n(x)$, find the value of $H_1(x)$.
13. Write the value of $Y_0^0(\Theta, \varphi)$.
14. Using $P_0(x) = 1$, $P_1(x) = x$. Find $P_2(x)$.
15. Write the value of $P_1(\cos \Theta)$.
16. Find the value of $P_2^2(\cos \Theta)$.
17. What is the value of $Y_{10}(\Theta, \varphi)$.
18. Check the relation $(2n + 3)xP_{n+1} = (n + 2)P_{n+2} + (n + 1)P_n$
19. What is the value of $erfc(x)$?
20. Show that $erf(\infty) = 1$.
21. show that $\beta(m+1,n) = m/(m+n)\beta(m,n)$.
22. Find the value of $\beta(3, 5)$.
23. Show that $erfc(x) = 1 - erf(x)$.
24. Write Legendre's duplication formula.
25. What do you mean by normal modes of vibration in a string ?

III. 2.5 mark questions.

1. State Dirichlet's conditions.
2. Write the Orthogonality relation of sine and cosine functions.
3. If $f(x) = k$ is a point of discontinuity then state the value of $f(x)$ at $x = k$.
4. If $f(x) = x^4$ in $(-2, 2)$, find the value of b_n.
5. Find the complex form of Fourier series for e^{ax} in $(-L, L)$.
6. Find the regular singular point of $2(1 - x)y'' - xy' + y = 0$.
7. Using Rodrigue formula, find the value of $P_2(x)$.
8. Show that $P_n(1) = 1$.
9. Showthat $P_n(-x) = (-1)^n P_n(x)$.
10. Write the value of $J_1(x)$.
11. Find the value of $x^3 + 2x^2 + 3x + I$ in terms of Legendre's polynomials. 12. Show that $(x^2 - 1)P'_n = n[xP_n - P_{n-1}]$
13. Show that $\beta(m, n) = \beta(n, m)$.
14. Check that $erf(x) + erfc(x) = 1$.

15. Show that $\beta(1, 2) + \beta(2, 1) = 1$.
16. Prove that erf(x) is an odd function of x.
17. Two parallel conducting plates are kept at potentials V_1 and V_2 respectively. Find the potential V between the plates if Laplace equation is satisfied.
18. Find the frequencies of first three harmonics if the wave velocity of the string is 100 m/s and length of the string is 1m.
19. Find the speed of transverse wave in a string of 1 mm² cross section under the tension 100N. ($g = 10$ m/s², density of material of wire is 10 gm/cm³).
20. Two strings A and B are made of the same material. The cross sectional area of A is twice that of B and tension on A is half that on B. Find the ratio of velocities of transverse waveson the two wires.

IV. 5 marks questions

1. Find the series of sines and cosines of multiples of x which represents f(x) in the interval

   -□□x□□   Where

   F(x) = $f(x) = \begin{cases} 0 & when & -\pi < x < 0 \\ \frac{\pi x}{4} & when & 0 < x \le \pi \end{cases}$

2. Expand the fourier series function $f(x) \;□X^2$in the interval □□□x□0and evaluate

   (i) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (ii) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

3. Derive Rodrigue's formula for legendre polynomial and find out the value of $p_1(x)$ and $p_2(x)$.

4. Expand $f(x) = \begin{cases} -sinx & when & -\pi \le x \le 0 \\ sinx & when & 0 \le x \le \pi \end{cases}$

5. Prove that $xH_{n-}{}^{1} = (n-1)H_{n-2} + \frac{1}{2}H_n$

6. Solve Legendre differential equation and prove that $P_n(x)$is a solution of the equation then $P_n(-x) = (-1)^n P_n(x)$ and $P_n^1(x) = (-1)^{n+1} P_n^1(x)$

7. Show that

8. Derive the general solution of Laplace equation in spherical coordinates.

9. Derive the solution of the wave equation of vibrating string tied at two ends given by the equation

10. A neutral conducting sphere is placed in an uniform electric field. Find the expression for new perturbed electrostatic potential.

11. Find the value of ∇^2 in cylindrical co-ordinates and obtain Laplace equation and solve it.

12. Define Error function and show that these functions can be written as incomplete Gamma function.

13. Solve the equation of motion of a vibrating stretched string.

14. Derive associated Legendre equation and reduce it to Legendre equation.

15. Define beta and gamma function. Hence find out the relation between them.