Karanjia Auto College,Karanjia,Mayurbhanj

CC-8

Mathematical Physics – III

I. One mark questions.

1. Find the complex conjugate of 3-4i.
2. Evaluate $\int_1^{1+i\pi} e^z \ dz$.
3. Find $\int_{\pi/4}^{\pi i/4} sec^2 \ zdz$.
4. Evaluate $\oint_c \ e^z \ dz \ where \ C \ is \ the \ unit \ circle$.
5. Find $\oint_c \ cos \ z$ dz where C is any simple closed path.
6. Evaluate $\int_0^{\pi} cos\left(\frac{z}{2}\right)$ dz.
7. Evaluate $\oint_c \frac{2z+1}{z(z+1)}$ dz where C is $|z|$ = ½.
8. Evaluate $\oint_c \frac{cos \ \pi z}{z-1}$ dz, where C is the circle $|z|$ = 3.
9. Find the order of zeros of f(z)=sin z.
10. The necessary condition for convergence of the Laplace transform is the absolute integrability of $f(t)e^{-st}$. The statement is
    (a) True (b) False (c) Sometimes true (d) Not related
11. Laplace transform of δ(t) is
    (a) 1 (b) 0 (c) ∞ (d) 2
12. Laplace transform of 2^t is
    (a) 1/In2 (b) 1/(s-In 2) (c) ln 2/(s-2) (d) In (s-2)
13. Laplace Transform of Unit Step Function is
    (a) 1 (b) 1/s (c) s (d) 2s
14. Inverse Laplace transform of $s/(2s^2-8)$ is
    (a) cosh 2t (b) $\frac{1}{2}$ cosh 2t (c) $\frac{1}{2}$sinh 2t (d) sinh 2t
15. If F(ω) is the Fourier Transform of f(x), then F[f(ax)] is
    (a) aF(ω) (b) (1/a)F(ω) (c) (1/a)F(ω) (d) (1/a)F(w/a)
16. If F(ω) is the Fourier Transform off(x), then F[f(x-a)] is (a) $e^{\omega a}$(b) $e^{i\omega a}$(c) $e^{-\omega a}$ (d) $e^{-i\omega a}$
17. The Fourier transform of a unit step function is given as (a) F (iω)= 1/iω (b) F(iω) =iω(c)F(iω)= i/ω (d) F(iω)= ω/i
18. Fourier transform ofδ(t) is given as (a) Zero (b)1 (c) 2πδ(ω) (d) πδ(ω)
19. Find the inverse Fourier transform of iω.
    (a) δ(t) (b) $\frac{d\delta(t)}{dt}$ (c) $\frac{1}{\delta(t)}$ (d) $\int \delta(t)dt$
20. The complex integral $\int_c \ tan\,(2\pi z)dz$, where C is the curve |z | = 1 is (a) 0 (b) -2πi (c) πi (d) 2πi

21. $\int_C \frac{dz}{(z-z_0)^2}$

(d) 1 , where C is any simple closed contour enclosing z_0 is equal to (a)2πi (b) 0 (c) $2\pi i z_0$

22. What is the valueof $\int_C \frac{dz}{(z^2+4)(z^2+4)}$ where C is |z-i|=2is (a)$\frac{\pi}{4}$ (b)$\frac{\pi}{8}$ (c)$\frac{\pi}{16}$(d)2π
23. A region which is not simply connected is called ________ region.(a) Multiple curve (b) Jordan connected (c) Multi connected (d) Connected curve
24. An integral curve along a simple closed curve is called a (a) Multiple curve (b) Jordan connected (c) Connected curve (d) Contour integral
25. If f(z) is analytic andf'(z) is continuous at all points inside and on simple closed curve C, then

(a) $_C f(z)dz=0$ (b) $_C f(z)dz\neq 0$ (c) $_C f(z)dz=1$(d) $_C f(z)dz\neq 1$

II. 1.5 mark questions.

1. Find the values of i^{53}.
2. Find the values of i^{100}.
3. Find the values of i^{i}.
4. Show that $(\cos 5\Theta - i \sin 5\Theta)^2(\cos 7\Theta + i \sin 7\Theta)^{-3}/(\cos 4\Theta - i \sin 4\Theta)^9(\cos \Theta + i \sin \Theta)^5=1$.
5. Define pole of complex function.
6. Determine the value of $L[e^{at}]$.
7. Determine the value of L[sin αt].
8. Determine the value of $L[t^a]$.
9. Determine the value of L[cos h at].
10. Find the finite sine transform of e^{ax}.
11. Find the finite sine transform of sin ax.
12. Find the Laplace transform of e^{kt}.
13. Find the Laplace transform of cos at.
14. Find the Laplace transform of sin at.
15. Find the inverse Laplace transform of $\frac{a}{s(s-a)}$.
16. Find the Laplace transformof $f(t) = 4t^2-3$.
17. Find the Laplace transformof $f(t) = 2t^{1/2}$.
18. State de Moivre’s theorem.
19. What is an analytic function ?
20. What is a simple and multiple curve ?
21. Define the limit and continuity of a complex function.
22. State Taylor series expansion.
23. Define Laplace transform of a function f(x).
24. Find the Laplace transform of the derivative of a function y = f(x).

III. 2.5 mark questions.

1. Express in the form of (x + iy) : 1/1+i
2. Express in the form of (x + iy) : $(\frac{1+i}{1-i})^2$
3. Find the complex conjugate of (2+4i)/(1-i)
4. Find the modulus of 1 + sin α + i cos α

5. Find the minimum positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$.
6. Find the values of x and y so that $z_1=z_2$ if $z_1=3x+5iy$, $z_2=2y+(3x+3)i$
7. Obtain Euler's formula for $e^{i\Theta}$.
8. Find the region of analyticity of f(z) = log z.
9. Find the Fourier transform of the Dirac delta function δ(x).
10. State and explain the 'Change of scale' property of Fourier transform.
11. Find Fourier transform of $\frac{1}{x^2+a^2}$.
12. Give the definition of Laplace transform.
13. State and prove Cauchy's integral theorem for a simple curve.
14. Prove Taylor series expansion.
15. Define inverse Fourier transform.
16. Discuss the linearity property of Fourier transform.
17. Discuss the shifting property of Fourier transform.
18. Find the Laplace Transform of integral of a function f(x). **IV. 5 marks questions**

1. State and prove de-Moiver's theorem for a positive and negative integer.
2. What is an analytic function? What are the necessary and sufficient conditions for the function f(z)=u+iv to be analytic at all points in a given region of R?
3. Write notes on Fourier sine and cosine transform.
4. Write a note on Fourier transform of derivatives. Define inverse Fourier transform.
5. Define Laplace transform of a function f(x). State and prove the change of scale of property and first shifting property of Laplace Transform.
6. A resistance R in series with inductance L is connected with e.m.fE(t). The current is given by $L\frac{di}{dt}$ +Ri=E(t). If the switch is connected at t=0 and disconnected at t=0, find the current I by Laplace Transform method.
7. Derive Cauchy-Riemann equations.
8. State and prove Taylors theorem.
9. State and prove Cauchy's Residue theorem.
10. State and prove convolution theorem.
11. State and prove that properties of Laplace transform.
12. Define periodic function and find out the Laplace transform of the Periodic function f(t).
13. Solve the differential equation of damped harmonic oscillator by applying Laplace transform.
14. State and prove Cauchy's integral formula.
15. State and prove Laurent series expansion.
16. State and prove Fourier integral theorem.
17. Obtain the Laplace transform of half wave rectifier function.