# 2020-21

***Time - 3 hours***

***Full Marks - 60***

*Answer **all groups** as per instructions.*
*Figures in the right hand margin indicate marks.*

## GROUP – A

1. Answer all questions or fill in the blanks as required. [1 × 8

(a) Write down the order and degree of differential equation

$$\frac{d^2y}{dx^2} + y = \tan x.$$

(b) If $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, then $\mathbf{A} \cdot \mathbf{C} =$ ______ .

(c) $\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} =$ ______ .

(d) If $r = \sqrt{x^2 + y^2}$, the value of

$$x\frac{\partial r}{\partial x} + y\frac{\partial r}{\partial y} = \text{\_\_\_\_\_\_} .$$

(e) The value of $\int_0^2 x^2\,\delta(x-3)\,dx =$ ______ .

(f) If $\mathbf{F}$ is a conservative field, then the value of $\nabla \times \mathbf{F} =$ ______

(g) The value of $\nabla(\mathbf{a}\cdot\mathbf{r})$ = ________ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(h) Write down the condition for **A** to be a solenoidal vector.

## GROUP – B

2. Answer any eight of the following questions. [1½ × 8

(a) What is linear and homogeneous differential equation ? Illustrate with an example.

(b) Show that the functions $e^x, e^{-2x}$ are linearly independent.

(c) Determine $\lambda$ such that $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \lambda\mathbf{j} + 3\mathbf{k}$ are coplanar.

(d) If **a** and **b** be two unit vectors and $\theta$ be the angle between them, then find the value of $\theta$ such that $\mathbf{a} + \mathbf{b}$ is a unit vector.

(e) Show that the differential equation

$$(x^2 + 2ye^{2x})dy + (2xy + 2y^2e^{2x})dx = 0 \text{ is exact.}$$

(f) Write down the gradient operator in spherical polar coordinates.

(g) Show that Dirac Delta function is an even function.

(h) State Gauss' divergence theorem.

(i) If $\mathbf{F} = (x^2 - y^2 + x)\mathbf{i} - (2xy + y)\mathbf{j}$, then find $\nabla \times \mathbf{F}$.

(j) Show that vector $\mathbf{A} = (x + 3y)\mathbf{i} - (y - 3z)\mathbf{j} + (x - 2z)\mathbf{k}$ is solenoidal.

## GROUP – C

3. Answer any eight of the following. [2 × 8

(a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0.$$

(b) Obtain Taylor series expansion of $f(x) = \sin x$ about $x = 0$.

(c) Find the value of $\lambda$, for differential equation

$$(xy^2 + \lambda x^2 y)dx + (x + y)x^2\, dy = 0 \text{ is exact.}$$

(d) Show that $\mathbf{a} \times (\mathbf{b} \times \mathbf{a}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{a}$.

(e) Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 5$ at the point (0, 1, 2).

(f) If $u = x^2 + y^2 + z^2$ and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, then find

$\nabla . (u\mathbf{r})$ in term of u.

(g) Evaluate : $\int_{-1}^{1} 9x^2\, \delta(9x + 1)dx$.

(h) If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$,

find the value of $\frac{\partial(u, v)}{\partial(x, y)}$.

(i) Find the value of $\mathbf{F} = 2z\mathbf{i} - 2x\mathbf{j} + y\mathbf{k}$ in term of cylindrical coordinates.

(j) State Green's theorem in the plane. How can you use it to find area ?

## GROUP – D

*Answer **any four** questions.*

4. Solve the differential equation [6

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x \sin x.$$

5. The pressure P at any point (x, y, z) in space is $P = 400xyz^2$. Find the highest pressure at surface of a unit sphere $x^2 + y^2 + z^2 = 1$. [6

6. A particle is moving in space. Express its position vector, velocity vector and acceleration vector in terms of spherical polar coordinates. [6

7. Solve : $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$. [6

8. Find the directional derivative of $\frac{1}{r}$ in the direction **r** where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. [6

9. Evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}\, ds$ where $\mathbf{F} = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$ and S is the surface of cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1. [6

10. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j}$ and C is the boundary of square in the plane z = 0 and bounded by the lines x = 0, y = 0, x = a and y = a. [6