No. of Printed Pages : 5

Sem-I-Math-GE-A$_1$
(Reg&Back)

2020-21

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Symbols used have their usual meaning.

GROUP – A

1. Answer all questions. [1 × 12

(a) Find the radius of curvature at any point of the curve

$s = c \log \sec \psi$.

(b) Find the asymptotes parallel to co-ordinate axes of the curve

$(x^2 + y^2)x - y^2a = 0$.

(c) Find the radius and centre of the sphere

$x^2 + y^2 + z^2 - 2x + 3y + 4z + 5 = 0$.

(d) Expand sin x by Maclaurin's series.

(e) Define Rolle's theorem.

(f) $\lim\limits_{x \to 1} \dfrac{1 + \log x - x}{1 - 2x + x^2}$; find the limit.

P.T.O.

(g) Find the limit $\lim_{(x, y) \to (2, 1)} \frac{\sin^{-1}(xy - 2)}{\tan^{-1}(3xy - 6)}$.

(h) If $f(x, y) = x^2 + y^2 + (x + y + 1)^2$, then find the value of $f_{xx}(x, y)$ and $f_{xy}(x, y)$ at $(0, 0)$.

(i) Find the value of x and y when $f_x = 0 = f_y$ for $f = x^2 + xy + y^2 + ax + by$.

(j) Write the solution of $x\,dy = 2\sqrt{y}\,dx$.

(k) Find the solution of the differential equation $y = xp - p^2$.

(l) Find the solution of $D^2y + 4y = 0$.

GROUP – B

2. Answer any eight of the following questions . [2 × 8

(a) Solve : $dx + (x + y)dy = 0$.

(b) Find the particular solution of $(D^2 + 4)y = \tan 2x$.

(c) Solve the equation : $y = 2px - p^2$.

(d) Find the radius of curvature of the curve at the origin

$$x^3 - 2x^2y + 3xy^2 - 4y^3 + 5x^2 - 6xy + 7y^2 - 8y = 0.$$

(e) Find the asymptotes of the Folium

$$x^3 + y^3 - 3axy = 0.$$

(f) Find the coordinates of the vertex of the cone

$$x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0.$$

(g) Expand the function $f(x, y) = e^{ax} \cos$ by in Maclaurin's theorem.

(h) Find $f_{yx}(0, 0)$ for the function

$$f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2} \quad , (x, y) \neq (0, 0)$$

$$= \quad 0 \quad , \text{otherwise.}$$

(i) Find the limit

$$\lim_{(x, y) \to (0, 0)} \frac{2xy^2}{x^4 + y^4} .$$

(j) Find the maximum value of the function

$$(x - 1)(x - 2)(x - 3).$$

GROUP – C

3. Answer <u>any eight</u> of the following questions. [3 × 8

(a) Find the limit $\lim_{x \to 0} \frac{(1 + x)^{\frac{1}{x}} - e}{x}$.

(b) Examine the function $\sin x + \cos x$ for extreme values.

(c) Discuss the continuity of the function at the origin, where

$$f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \quad , \text{if } x^2 + y^2 \neq 0$$

$$= 0 \quad , \text{if } (x, y) = (0, 0).$$

(d) If $z = f(x^2y)$, then find dz.

(e) Solve : $(2x + y + 3)\frac{dy}{dx} = x + 2y + 3$

(f) Solve : $\frac{dy}{dx} + xy = x^3y^3$.

(g) Solve : $(D^5 - D)y = 12e^x + 8 \sin x - 2x$

(h) Find the equation of the cylinder whose generators are parallel to the line $x = y = z$ and whose guiding curve is the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1, z = 1.$$

(i) Rectify the curve

$$x = a(\theta + \sin \theta), y = a(1 - \cos \theta).$$

(j) Find the radius of curvature at any point of the curve

$$x = \frac{a \cos t}{t}, y = \frac{a \sin t}{t}.$$

(k) Evaluate the integral

$$\int\int_R (x^2 + 2y)dx\, dy ; R = [0, 1 ; 0, 2].$$

GROUP – D

*Answer **any four** questions.*

4. Find the equation of the enveloping cone of the sphere [7

$x^2 + y^2 + z^2 - 2x + 4z - 1 = 0$ with vertex at (1, 1, 1).

5. Trace the curve $ay^2 = x(a^2 - x^2)$. [7

6. Solve the differential equation [7

$x^2(y - px) = p^2y$; $p = \frac{dy}{dx}$.

7. Solve the differential equation [7

$(D^2 - 4D + 3)y = e^x \cos 2x + \cos 3x$.

8. Solve using the method of variation of parameter [7

$(D^2 + 4D + 5)y = e^{-2x} \sec x$.

9. Find all the maximum and minimum values of the function [7

$f(x, y) = x^2 + xy + y^2 + ax + by$.

10. Expand $x^2y + 3y - 2$ is power of $x - 1$ and $y - 2$. [7

11. Show that, using Taylor's theorem, [7

$$x - \frac{x^3}{6} < \sin x < x - \frac{x^3}{6} + \frac{x^5}{120}, \text{ for all } x > 0.$$