2022

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

GROUP - A

1. Answer <u>all</u> questions. [1 × 8

   (a) The solution of the differential equation $dy = (1 + y^2)dx$ is
   _____.

   (b) Write the degree of the differential equation $y = x\dfrac{dy}{dx} + \left(\dfrac{dy}{dx}\right)^{-2}$.

   (c) What do you mean by radioactive materials ?

   (d) What is SIR Model ?

   (e) Write a word equation of simple exponential decay mode.

   (f) The solution of differential equation $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$ is
   _____.

(g) Write an example of linear homogeneous differential equation of second order.

(h) The ordinary differential equation $\dfrac{d^2u}{dx^2} - 2x^2u + \sin x = 0$ is _____. (Choose the correct answer.)

    (i) Linear and homogeneous

    (ii) Linear and nonhomogeneous

    (iii) Nonlinear and homogeneous

    (iv) Nonlinear and nonhomogeneous

GROUP - B

2. Answer <u>any eight</u> questions.   [1½ × 8

(a) Find the general solution of $\left(xy\dfrac{dy}{dx} - 1 \right) = 0$.

(b) Find the integrating factor of the differential equation

$$\dfrac{dy}{dx}(x \log x) + y = 2 \log x .$$

(c) Solve the initial value problem $\dfrac{dy}{dx} + y = 2xe^{-x}$, $y(0) = 2$, given the General Solution as $y = (x^2 + c)e^{-x}$.

(d) Write a word equation of lake pollution model.

(e) What do you mean by radioactive materials ?

(f) Find the general solution of the differential equation $y'' + 8y = 0$.

(g) Solve $(D^2 - 2D + 1)y = e^{-x}$.

(h) Solve $y'' + 6y' + 9y = 0$.

(i) Write word equation of influenza outbreak model.

(j) Determine possible direction of phase plane trajectories in the phase plane.

GROUP - C

3. Answer <u>any eight</u> questions.   [2 × 8

(a) Find general solution of $\dfrac{dx}{dy} = (1 + x^2)(1 + y^2)$.

(b) Solve : $(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$.

(c) Find an integrating factor for the differential equation

$$(x - \ln y)\frac{dy}{dx} = -y\ln y$$

(d) Write word equation for density dependent growth of population.

(e) Find the residence time of exponential decay compartmental model.

(f) Find the general solution of the differential equation $2y'' - 4y' + 8y = 0$.

(g) Solve $(D^2 + 3D - 10)y = 6e^{4x}$.

(h) Find the particular integral of $(D - 2)^2 y = e^{2x}$.

(i) Find an expression for the line for the population to double in size.

(j) Write compartmental diagram for simple battle model.

GROUP - D

4. Answer <u>any four</u> questions.   [6 × 4

(a) Verify the following equations are homogeneous and solve them $x^2 \dfrac{dy}{dx} - 3xy - 2y^2 = 0$.

(b) Solve : $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$

(c) How long will it take for the lake pollution level to reach 5% of its initial level if only fresh water flows into the lake ?

(d) Taking $r = 1, k = 10, h = \dfrac{9}{10}$ and $X(0) = x_0$ in the initial growth with harvesting Model, investigate the behaviours of the solution of the logistic differential equation.

(e) Find the P.I. of $(D^3 + 1)y = e^x \cos x + \sin 3x$.

(f) Find the particular integral of the equation $(D^2 + 2D + 1)y = e^x \sin^{-1}x$ by the method of variation of parameter.

(g) Find equilibrium points of the coupled system

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -x.$$