2022

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.

GROUP – A

1. Fill in the blanks. (<u>all</u>)   [1 × 8

    (a) The value of i^{103} is _____ .

    (b) The value of $(1 + i) / (3 + 4i)$ is _____ .

    (c) The complex conjugate of $(2 + 3i) / (1 - i)$ is _____ .

    (d) The Fourier Cosine transform of e^{-ax} is _____ .

    (e) The Fourier Sine transform of $1/x$ is _____ .

    (f) The Laplace transform of $1/\sqrt{t}$ is _____ .

    (g) The Laplace transform of a unit step function with step at point a is _____ .

    (h) The Laplace transform of $\cos(t)$ is _____ .

GROUP – B

2. Answer <u>any eight</u> of the following questions within two to three sentences each. [1½ × 8

(a) Find the Fourier transform of $e^{-|t|}$.

(b) Define the Fourier transformation.

(c) State the convolution theorem for Laplace transform.

(d) Find the polar form of Complex number $z = -\sqrt{3} - i$.

(e) Write down Cauchy's integral formula.

(f) Evaluate the Laplace transform of $\sin(2t)/t$.

(g) Find the value of i^{333}.

(h) Define essential singularity.

(i) State first shifting property of inverse Laplace transform.

(j) What is analytic function ?

GROUP – C

3. Answer <u>any eight</u> of the following questions within 75 words each. [2 × 8

(a) Express $\dfrac{i}{-3 - 3i}$ in the standard form x + iy.

(b) Prove that $2x + ixy^2$ is not analytic.

(c) Find the Laplace transform of e^{at}.

(d) Define essential singularity.

(e) Prove shifting property of Fourier transform.

(f) Write change of scale of property of Fourier transform.

(g) Define removable singularity.

(h) Find the three cube roots of unity ?

(i) Write the polar form of Cauchy-Riemann equations.

(j) State Taylor's theorem for complex analytic functions.

GROUP – D

*Answer **any four** questions within 500 words each.*

4. Prove that the function u = 2x(1 – y) is harmonic. Find a function v such that f(z) = u + iv is analytic. [6

5. Obtain the polar form of Cauchy-Riemann equations. [6

6. Evaluate the contour integration $\int_0^{2\pi} \frac{\cos(3\theta)\, d\theta}{(5 - 3\cos\theta)}$. [6

7. Evaluate the contour integration $\int_0^\infty \frac{\sin(x)dx}{x}$. [6

8. State and prove convolution theorem in integral transform. [6

9. Using Fourier Transforms, explain one dimensional heat flow. [6

10. Evaluate using Laplace transforms the following integral :　　[6

$$\int_0^\infty te^{-3t}\sin t\, dt$$

11. Using Laplace transform, solve the given coupled differential equations :　　[6

$$\frac{dx}{dt} = 2x - 3y$$

$$\frac{dy}{dt} = y - 2x$$

subject to initial conditions $x(0) = 8$, $y(0) = 3$.