No. of Printed Pages : 6

Sem-V-Math-CC-11
(Reg&Back)

2023-24

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Part of each question should be answered continuously.
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.
The symbols used have their usual meanings.

GROUP – A

1. Answer all questions. [1 × 12

(a) Let $f(x, y) = \sqrt{9 - x^2 - 4y^2}$. Find domain and range of f.

(b) Determine f_x and f_{yx} in $f(x, y) = \sin(x^2y)$.

(c) Find the total differential of the function

$$f(x, y, z) = Z^2 \sin(2x - 3y)$$

(d) Show that a function $f(x, y) = e^x . \cos y$ is harmonic.

(e) Find the divergence of **F** at (3, 2, 1) when

$$\mathbf{F}(x, y, z) = xyz\mathbf{i} + y\mathbf{j} + x\mathbf{k}.$$

(f) What is the dominance rule in a triple integral ?

P.T.O.

(g) Find the curl of **F** at $(\frac{\pi}{4}, \pi, 0)$ when

$$\mathbf{F}(x, y, z) = (\cos y)\mathbf{i} + (\sin y)\mathbf{j} + \mathbf{k}.$$

(h) Evaluate the triple integral $\int_1^4 \int_{-2}^3 \int_2^5 dx\, dy\, dz.$

(i) Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ where $x = u + 2v$ and $y = e^u \cdot \cos v$.

(j) Find the conservative field in a region D for a scalar function $f = x^2 y$.

(k) Define independent of path in a region D.

(l) State the divergence theorem.

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Discuss the limit of function

$$\lim_{(x, y) \to (0, 0)} \frac{x^2 y}{x^4 + y^2}$$

(b) Verify the heat equation for the function $T(x, t) = e^{-t} \cos \frac{x}{c}$.

(c) Find the gradient of $g(x, y, z) = x \cdot e^{y + 3z}$.

(d) State Fubini's theorem over a rectangular region.

(e) Find the area of a region D between $y = \cos x$ and $y = \sin x$ on the interval $0 \le x \le \frac{\pi}{4}$ using a double integral

(f) Evaluate $\int_0^1 \int_x^1 e^{y^2}\, dy\, dx$ by reversing the order.

(g) Compute $\iiint_D xyz\, dv$, where D is the tetrahedron with vertices (0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1).

(h) Find the equation in cylindrical co-ordinates for the elliptic paraboloid $z = x^2 + 3y^2$.

(i) State Stoke's theorem.

(j) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by using Green's theorem.

GROUP – C

3. Answer any eight questions. [3 × 8

(a) Find the equation of tangent plane for a surface

$$z = \tan^{-1}\left(\frac{y}{x}\right) \text{ at } P_0\left(1, \sqrt{3}, \frac{\pi}{3}\right).$$

(b) Find the directional derivative of

$$f(x, y) = \log(3x + y^2) \text{ at } P(0, 1) \text{ in the direction of } \mathbf{v} = \mathbf{i} - \mathbf{j}.$$

(c) Find a vector that is normal to the level surface

$$x^2 + 2xy - yz + 3z^2 = 8 \text{ at } P_0 (2, 2, -1).$$

(d) Find all relative extrema and saddle points of the function

$$f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

(e) Compute $\iint_R x^2y \, dA$ where R is a rectangle, $1 \le x \le 2, 0 \le y \le 1$.

(f) Find the volume of the solid bounded above by the plane $z = y$ and bounded by below in the xy-plane in the first quadrant of the disk $x^2 + y^2 \le 1$.

(g) Find the volume of the tetrahedron T bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0$, $y = 0$ and $z = 0$.

(h) Find the volume of the sphere of radius R using triple integration.

(i) Evaluate the line integral $\int_C \frac{y^2}{x^3} dx$ over the curve

$C : x = 2t, y = t^4, 0 \le t \le 1$.

(j) Evaluate $\iint_S \mathbf{F} . \mathbf{N} \, ds$, where $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + x^3y^3\mathbf{k}$ and S is the surface of the tetrahedron bounded by the plane $x + y + z = 1$ and coordinate planes with $\hat{N}$.

GROUP – D

4. Answer any four questions. [7 × 4

(a) Find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$; the half-cone $z = \sqrt{x^2 + y^2}$; xy-plane.

(b) Compute the double integral $\iint_D \left(\frac{x-y}{x+y}\right)^4 dx\, dy$, where D is the rectangular region bounded by the line $x + y = 1$ and the coordinate axes.

(c) Let S be the portion of the plane $x + y + z = 1$ that lies in the first octant and let C be the boundary of S. Verify Stoke's theorem for the vector field $\mathbf{F} = -\frac{3}{2}y^2\mathbf{i} - 2xy\mathbf{j} + yz\mathbf{k}$.

(d) Show that the vector field $\mathbf{F} = (20x^3z + 2y^2, 4xy, 5x^4 + 3z^2)$ is conservative in $\mathbb{R}^3$ and then find a scalar potential function for $\mathbf{F}$.

(e) For the function $f(x, y) = \sqrt{|xy|}$, show that f_x and f_y exist at (0, 0) but f is not differentiable at (0, 0)

(f) Find the equation of tangent plane and normal line to the surface $x^2y + y^2z + z^2x = 5$ at (1, –1, 2).

(g) Find the absolute extrema of the function $f(x, y) = e^{x^2 - y^2}$ over the disk $x^2 + y^2 \le 1$.

(h) Evaluate $\iint\limits_D \frac{1}{x}.dA$,

where D is the region that lies inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1 + \cos\theta$.