No. of Printed Pages : 6

Sem-V-Math-DSE-II
(Reg&Back)

2023-24

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Part of each question should be answered continuously.
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.
The symbols used have their usual meanings.

GROUP – A

1. Fill in the blanks. (all) [1 × 12

(a) The relation between the probability density function f(x) and probability distribution function F(x) is ________ .

(b) If X has the variance 2, then Var(–2X + 6) is ________ .

(c) The distribution in which mean and variance are equal is ________ .

(d) If P(A) = 0.4, P(B) = 0.5 and $P(A \cap B) = 0.2$, then $P(B/A)$ is ________ .

(e) The interval in which the probability of an event E lies is ________ .

(f) The sample mean $(\overline{X})$ of random variables $X_1, X_2, X_3, \ldots\ldots, X_n$ is ________ .

(g) If X is a continuous random variable, then expected value of X = ________ .

(h) A bag contains 4 red, 5 blue and 3 green balls. If two balls are drawn at random, then the probability that both are red is ________ .

(i) A population has N items. Sample of size n are selected without replacement. Then the number of possible samples is ________ .

(j) The moment generating function of the normal distribution is $M_X(t) =$ ________ .

(k) Two events A and B are independent if $P(A \cap B) =$ ________ .

(l) If F(x) is the cumulative distribution function of a discrete random variable X, then $F(\infty) =$ ________ .

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) If A and B are any two events in sample space S, then show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(b) What is random sampling ? Explain with an example.

(c) Define Chi-square distribution.

(d) Define student's t-distribution.

(e) If the probability density function of a random variable X is given by

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{x}} & \text{for } 0 < x < 4 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the value of k.

(f) If the joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{4}(2x + y) & \text{for } 0 < x < 1,\ 0 < y < 2 \\ 0 & \text{elsewhere,} \end{cases}$$

find the marginal density of X.

(g) Write the mean and variance of gamma distribution.

(h) Prove that $Cov(X, Y) = Cov(Y, X)$ for continuous random variables X and Y.

(i) Find the expected value of the random variable whose probability density function is given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \le x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

(j) Define standard normal distribution.

GROUP – C

3. Answer any eight questions. [3 × 8

(a) Prove that

$$Cov(X, Y) = E(XY) - E(X) . E(Y).$$

(b) Find the mean of binomial distribution.

(c) Let X be a random variable with cdf F(x). Show that for $a < b$,

$$P(a < X \leq b) = F(b) - F(a).$$

(d) If three dice are thrown simultaneously, then find the probability that the sum of the digits is 16.

(e) The probability density of a random variable Z is given by

$$f(z) = \begin{cases} kze^{-z^2} & , \text{for } z > 0 \\ 0 & , \text{for } z \leq 0, \end{cases}$$

then find the value of k.

(f) A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find the value of k. Also find $P(X \leq 4)$.

(g) If the joint probability density of X, Y and Z is given by

$$f(x, y, z) = \begin{cases} \frac{1}{3}(2x + 3y + z) & \text{for } 0 < x < 1, 0 < y < 1, 0 < z < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

then find $P\left(X < \frac{1}{2}, Y < \frac{1}{2}, Z < \frac{1}{2}\right)$.

(h) Find the moment generating function of negative binomial distribution.

(i) If a random sample of size 'n' is selected without replacement from the finite population that consists of the integers 1, 2, 3, ..., N, show that the variance of $\overline{X}$ is $\frac{(N+1)(N-n)}{12n}$

(j) If A and B are two events such that

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$, then find

$P(A/B)$, $P(A \cup B)$ and $P(A^C \cap B^C)$.

GROUP – D

4. Answer any four questions. [7 × 4

(a) If the probability density of X, Y and Z is given by

$$f(x, y, z) = \begin{cases} kxy(1-z) & \text{for } 0 < x < 1, 0 < y < 1, 0 < z < 1, \\ & x + y + z < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

then find the value of k.

(b) If the joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{3}(x + y) & \text{for } 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

find the variance of $W = 3X + 4Y - 5$.

(c) Show that the mean and variance of beta-distribution are

$$\mu = \frac{\alpha}{\alpha + \beta} \text{ and } \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)} \text{ respectively.}$$

(d) Show that the total area under the bell shaped curve of normal distribution is unity.

(e) If the regression of Y on X is linear, then show that

$$\mu_{y/x} = \mu_2 + \rho \frac{\sigma_2}{\sigma_1}(x - \mu_1).$$

(f) Find the moment generating function of binomial distribution. Also use it to find the mean and variance of binomial distribution.

(g) State and prove Bayes' theorem.