2023-24

Time - 3 hours

Full Marks - 80

Answer all groups as per instructions.
Part of each question should be answered continuously.
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.
The symbols used have their usual meanings.

GROUP – A

1. Answer all questions and fill in the blanks as required.   [1 × 12

    (a) Define a line segment.

    (b) What is the drawback of Graphical method ?

    (c) Define artificial variable.

    (d) How one can say that in a Simplex method, there is an alternate solution ?

    (e) If the primal LPP has an infeasible solution, then what can you say about its dual solution ?

[2]

(f) In any of the variable in a primal is unrestricted in sign, then the corresponding dual constraint is _____ in sign.

(g) When a transportation problem is said to be unbalanced ?

(h) When degeneracy occurs in a transportation problem ?

(i) When we can say that a transportation problem under test is optimal ?

(j) Define two persons zero-sum game.

(k) What is the meaning of strategy of a game ?

(l) An assignment problem can be solved only when _____ .

GROUP – B

2. Answer _any eight_ of the following questions.   [2 × 8

(a) Write the general form of an LPP.

(b) Define convex combination.

(c) Define the optimal solution in an assignment problem.

(d) Define an assignment problem.

(e) Write the canonical form of maximization type LPP.

(f) Prove that dual of dual is primal.

[3]

(g) Find the saddle point of the game having payoff matrix as :

Player Y

$$\begin{array}{c} \\ \text{Player X} \end{array} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ \begin{bmatrix} -3 & -2 & 6 \\ 2 & 0 & 2 \\ 5 & -2 & -4 \end{bmatrix} \end{array}$$

(h) Make the following unbalanced transportation problem to a balanced transportation problem :

Destination

	D_1	D_2	D_3	D_4	Supply
O_1	–	–	–	–	70
O_2	–	–	–	–	55
O_3	–	–	–	–	70
Demand	85	35	50	45	

(Origin)

(i) Write the dual of the following primal :

Maximise $Z = 6x_1 + 8x_2$

subject to $5x_1 + 2x_2 \leq 20$

$x_1 + 2x_2 = 10$

and $x_1, x_2 \geq 0.$

P.T.O.

(j) Explain the dominance property in game theory.

GROUP – C

3. Answer <u>any eight</u> questions.

$[3 \times 8$

(a) Solve graphically :

Maximise $Z = 2x_1 + 3x_2$

subject to $\quad x_1 + 2x_2 \leq 30$

$2x_1 + x_2 \leq 30$

and $\quad x_1, x_2 \geq 0.$

(b) Mention three differences between assignment problem and transportation problem.

(c) Find the IBFS by Vogel's Approximation method :

6	4	1	5	14
8	9	2	7	16
4	3	6	2	5
6	10	15	4	

(d) A hyperplane is given by the equation

$3x_1 + 2x_2 + 4x_3 + 7x_4 = 8.$

Determine in which half-spaces do the point $(-6, 1, 7, 2)$ lie.

(e) Prove that a hyperplane is a convex set.

(f) Write down the matrix form of primal-dual problem.

(g) Write down the dual of the following primal :

Minimise $Z = x_1 + x_2 + x_3$

subject to $\quad x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \leq 3$

$2x_2 - x_3 \geq 4$

$x_1, x_2 \geq 0, x_3$ is unrestricted.

(h) Explain the term pure strategy and mixed strategy.

(i) Use Simplex method to find the inverse of the matrix $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$.

(j) For what value of λ, the game with the following pay-off matrix is strictly determinable :

Player B

		B_1	B_2	B_3
	A_1	λ	6	2
Player A	A_2	-1	λ	-7
	A_3	-2	4	λ

GROUP – D

4. Answer <u>any four</u> questions. [7 × 4

(a) Solve the following LPP :

Maximise $Z = 3x_1 + 2x_2 + 5x_3$

subject to $\quad x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

and $\quad x_1, x_2, x_3 \geq 0$

(b) Given the cost matrix of an assignment problem. Find the optimal assignment.

Jobs

11	17	8	16	20
9	7	12	6	15
13	16	15	12	16
21	24	17	28	26
14	10	12	11	15

Persons

(c) Solve the following game by linear programming technique :

Player B

Player A $\begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{pmatrix}$

(d) Find the optimal solution of the following LPP using duality theory :

Maximise $Z = 2x_1 + x_2$

subject to $\quad x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

and $\quad x_1, x_2 \geq 0$

(e) Using dual-Simplex method, solve the following LPP :

Maximise $Z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

subject to $\quad 5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$

$x_2 + 5x_3 - 6x_4 \geq 10$

$2x_1 + 5x_2 + x_3 + x_4 \geq 8$

and $\quad x_1, x_2, x_3, x_4 \geq 0$

(f) Solve the following assignment problem :

Task	Men			
	1	2	3	4
I	15	14	12	16
II	23	22	25	24
III	31	34	32	33
IV	21	22	44	53

(g) Solve the 3 × 2 game graphically :

Player B

		I	II
	I	3	5
Player A	II	3	5
	III	11	2