No. of Printed Pages : 5

Sem-VI-Math-CC-13
(Reg&Back)

2023

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.
The symbols used have their usual meanings.

GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) If the amplitude of the complex number z be θ, then amplitude of iz is __________ .

(b) If $z = x + iy$, then $z\bar{z}$ = _________ .

(c) Exp $(2 \pm 3i\pi)$ = (Indicate the correct answer.)

(i) e^{-2} (ii) $-e^{2}$

(iii) $e^{\pm 3i}$ (iv) None of these

(d) In the Argand plane, $|z+i| = |z-i|$ represents a straight line. (Write true / false.)

(e) The function $f(z) = |xy|^{\frac{1}{2}}$ is analytic at $z = 0$. (Write true / false.)

(f) An analytic function with constant modulus is (Indicate the correct answer.)

(i) variable

(ii) constant

(iii) may be variable or constant

(iv) none of these

(g) The function $f(z) = \dfrac{e^z}{(z-1)^3}$ has a pole of order ________ at $z = 1$.

(h) The zeros of an analytic function are isolated. (Write true / false.)

(i) Residue of $\cos\left(\dfrac{1}{z-2}\right)$ at $z = 2$ is ________ .

(j) The real part of $(i)^i$ is ________ .

(k) The zeros of tan z are ________ .

(l) Residue of $\dfrac{1}{z^3 - z^5}$ at $z = \pm 1$ is ________ .

GROUP – B

2. Answer <u>any eight</u> of the following questions. [2 × 8

(a) If $z = 1 + i\sqrt{3}$, then find $|\arg z| + |\arg \bar{z}|$

(b) Define holomorphic function.

(c) Prove that $\bar{z}$ is not differentiable.

(d) Evaluate : $\int_{-i}^{i} \frac{dz}{z}$.

(e) Find exponential form of $z = \frac{i}{1+i}$.

(f) Express $z = 2i$ in its polar coordinate form.

(g) Fins radius of convergence of the power series

$$\sum \frac{n\,!}{n^2} z^n.$$

(h) Define Residue of f(z) at infinity.

(i) Define removable singularity.

(j) Define zero of an analytic function.

GROUP – C

3. Answer any eight questions. [3 × 8

(a) Prove that $|z|^2$ is continuous everywhere, where z is a complex number.

(b) Find the modulus and argument of the complex number

$$z = \frac{2+i}{4i + (1+i)^2}.$$

(c) Find the square root of $z = 3i$.

(d) Evaluate $\int_{0}^{\pi/4} e^{-2it}\, dt$.

(e) Find the pole(s) and its order of the function

$$f(z) = \frac{z + 1}{z^3(z^2 + 1)}.$$

(f) Prove that $f(z) = z^2$ is analytic by the help of Cauchy Riemann equation.

(g) State Schwarz reflection principle.

(h) Determine $\log(1 + i)$.

(i) If $z = a$ is a pole of $f(z)$, then show that $|f(z)| \to \infty$ as $z \to a$.

(j) Prove that $\frac{1}{z^4 + 2z^2 + 1}$ has two double poles.

GROUP – D

4. Answer any four questions. [7 × 4

(a) Show that $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conjugate and find the corresponding analytic function $f(z)$ in terms of z.

(b) Prove that the function $e^x(\cos y + i \sin y)$ is holomorphic and find its derivative.

(c) Evaluate $\oint_C \frac{z^2}{z^2-1}\,dz$, C : $|z+1| = \frac{3}{2}$ counter clockwise.

(d) State and prove Cauchy's integral formula.

(e) Evaluate $\int_C \frac{e^{2z}\,dz}{(z+1)^4}$ where C is $|z| = 3$.

(f) State and prove Morera's theorem.

(g) Find the value of

$$\int_0^{1+i} (x - y + ix^2)dz$$

along the straight line joining $z = 0$ to $z = 1 + i$.