No. of Printed Pages : 5

Sem-VI-Math-DSE-3
(Reg&Back)

2023

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.
The symbols used have their usual meanings.

GROUP – A

1. Answer all questions. [1 × 12

(a) Write the equation of the osculating plane at any point P(t) in parametric form.

(b) Define curvature and torsion of a curve at a point.

(c) What are fundamental planes ?

(d) Define osculating sphere.

(e) Define involute of a given curve.

(f) What is developable surface ?

(g) Write the fundamental magnitudes of 1st order and 2nd order.

P.T.O.

(h) Define normal curvature.

(i) Define Geodesic curvature.

(j) Write the differential equation to the geodesic of a surface $z = f(x, y)$.

(k) Write the equation lines of curvature at any point $P(u, v)$ for the surface $r = r(u, v)$.

(l) Write the equation of surface of revolution.

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Define radius of curvature and radius of torsion.

(b) State Serret-Frenet's formula.

(c) Define a circular helix.

(d) State the conditions for which $LN - M^2 > 0$. (For a point on a surface)

(e) Write the value for normal curvature at any point of surface $r = r(u, v)$.

(f) Asymptotic lines are orthogonal if and only if $EN - 2FM + GL = 0$. Is it true ?

(g) Write the condition for a surface to be developable.

(h) Write the equation of tangent plane to the surface $z = f(x, y)$.

(i) Parametric curve are conjugate directing if and only if $M = 0$. Is it true ?

(j) State osculating plane, normal plane and rectifying plane.

GROUP – C

3. Answer any eight questions. [3 × 8

(a) Find the equation of osculating plane at (1, 0, 2) if $x = u$, $y = 1 - u$, $z = u(1 + u)$.

(b) Find the curvature of a circle of radius a.

(c) Find the equation of the tangent plane at any point of

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(d) Find the characteristic lines and the envelope of the family $(x - a)^2 + y^2 + z^2 = 1$, a is a parametric.

(e) Prove that the surface $x^2 + y^2 - z^2 = 0$ is developable.

(f) Find the fundamental magnitudes of 1st order of the surface

$$r(u, v) = u \cos v i + u \sin v j + avk.$$

(g) Show that the helices on a surface of a cylinder are geodesics.

(h) Give the statement of Meusnier theorem.

(i) A necessary and sufficient condition that a curve be a straight line is that $k = 0$.

(j) The distance between corresponding points of two Betrand curves is constant. Prove it.

GROUP – D

4. Answer any four questions. [7 × 4

(a) The necessary and sufficient condition for a curve to be helix is that, the ratio of the curvature and the torsion of the curve is constant. Prove it.

(b) Prove that

$$x'''^2 + y'''^2 + z'''^2 = \frac{1}{\rho^2\sigma^2} + \frac{1+\rho'^2}{\rho^4}$$

where ' ′ ' denotes derivative with respect to S.

(c) Find the envelope of $3a^2x - 3ay + z = a^3$ and show that its edge of regression is the curve of intersection of the surface

$y^2 = zx$ and $xy = z$.

(d) Derive the condition that the quadratic differential equation

$$Pdu^2 + 2Q\,du\,dv + Rdv^2 = 0$$

represents orthogonal families of curves.

(e) Find the asymptotic lines of surface

$$x = 3u(1 + v^2) - u^3,\ y = 3v(1 + u^2) - v^3,\ z = 3(u^2 - v^2).$$

(f) Find the Fundamental magnitude of 2nd order and 2nd fundamental form of the sphere $x^2 + y^2 + z^2 = a^2$.

(g) Find the principal curvatures at a point $u = 0$, $v = 1$ on the surface $x = u \cos v$, $y = u \sin v$, $z = v$.