2020-21

Time - 3 hours
Full Marks – 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.

Group-A

1. Answer <u>all</u> questions or fill in blanks as required. [1x12]

   a) The value of $\lim_{x \to 1} \frac{\log x}{x-1}$ is __________.

   b) What is maximum value of $\cos(\cos(\sin x))$?

   c) Which of the following is not an indeterminate form?

   (Choose the correct answer)

   i) $\infty + \infty$ ii) $\infty - \infty$ iii) ∞/∞ iv) $0 \times \infty$

   d) Give an example of a function which is Riemann integrable but not monotonic.

   e) If f is integrable on [a, b], $f^2$is also integrable. (True / False).

   f) Give an example of a function which is not integrable but |f| is integrable.

   g) What is improper integrable of first kind?

   h) A proper integral is always convergent. (True/False)

   i) $\int_0^\infty e^{-cx}$dx is equal to: (Choose the correct answer)

   i) $c^n\Gamma(n)$ ii) $c^{n-1}\Gamma(n)$ iii) $\frac{(\Gamma(n))}{c^n}$ iv) $\frac{(\Gamma(n))}{c^{n-1}}$

   j) When a sequence of function $\{f_n\}$ is said to be convergent on $S \subseteq D$?

   k) If $\Sigma\, a_n z_0^n$ is convergent, then when $\Sigma\, a_n z^n$ is absolutely convergent?

   l) What is the radius of convergence of $\Sigma\, n!\, z^n$?

Group-B

2. Answer any eight of the following questions within two or three sentences each. [2x8]

   a) Evaluate $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$.

   b) Verify mean value theorem for f(x) = log x in [1, e].

   c) Prove β(m, n) = β (n, m).

   d) Evaluate $\lim_{x \to \infty} \frac{\log x}{x^m}$ (m > 0).

   e) Show that maximum value of $\left(\frac{1}{x}\right)^x$ is $(e)^{\frac{1}{e}}$.

   f) Show that for a > -1,

   $$\lim_{n \to \infty} \frac{1}{n^{a+1}} \sum_{k=1}^{n} k^a = \frac{1}{a+1}$$

   g) Test the convergence of the integral $\int_0^\infty \frac{1}{\sqrt{x^2+1}}$ dx.

   h) Prove that the sequence (f_n) where $f_n(x) = \frac{1}{x+n}$ converges pointwise to 0.

   i) Test the uniformly convergence of the sequence

   $f_n(x) = \frac{nx}{1+n^2x^2}$ $x \in [0, \infty]$

   j) Find the radius of convergence of the power series

   $1 + \frac{x}{2} + \left(\frac{x}{4}\right)^2 + \left(\frac{x}{2}\right)^3 + \left(\frac{x}{4}\right)^4 + \ldots\ldots\ldots$

GROUP-C

3. Write notes on any eight of the followings within 75 words: [3x8]

   a) Evaluate $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$.

   b) Find the extreme value of the function f(x)= sin x+$\frac{1}{2}$ sin 2x+ $\frac{1}{3}$sin 3x $\forall$ [0, π].

c) Using Taylor' theorem, find a polynomial of f(x) of degree 2 which satisfies f(1) = 2, f'(1)=-1 and f"(1)=2.

d) Prove that a constant function on [a, b] is integrable.

e) Prove that every monotonic function on [a, b] is integrable.

f) Show that $\int_0^\infty \frac{x^2}{(1+x^2)^2}$ dx is convergent.

g) Prove that $\Gamma(n+\frac{1}{2})=\frac{(2n)!\sqrt{\pi}}{4^n.n!}$, n= 0, 1, 2.

h) Let f: R→ R be uniformly continuous in R and $f_n(x) = f(x+\frac{1}{n})$ for all x ∈ R. Prove that (f_n) converges uniformly to f on R.

i) Prove Abel's theorem.

j) Determine the interval of uniform convergence of the series

$$\sum_{n=1}^{\infty} k^a = \frac{x^n{}^n}{n^n}$$

GROUP- D

4. Answer any four questions within 500 words each. [7x4]

a) Prove that $1 - \frac{1}{2}x^2 \leq \cos x,\ x \in R$.

b) Expand sin x in powers of $(x - \frac{\pi}{4})$ with the help of Taylor's theorem.

c) Prove that $x^2$is integrable on any interval [0, k].

d) If f ∈ C[0, 1], show that $\lim_{x \to 1} \int_0^1 (n+1)x^n f(x)dx = f(1)$

e) Prove that $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots$

f) Prove that $\int_0^\infty e^{-cx^2}$dx= $\frac{1}{2}\sqrt{\pi}$.

g) Show that

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots.., -1 \leq x \leq 1.$$

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