2020-21

Time - 3 hours
Full Marks – 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.

Group-A

1. Answer all questions or fill in blanks as required. [1x12]
   a) Give an example of a non-Abelian group.
   b) Write the order of the group U (10).
   c) Does the set {0, 1, 2, 3} form a group under multiplication modulo 4?
   d) Define an Abelian / Commutative group.
   e) What is the order of the group S_3?
   f) If H and K are two subgroups of a group G, then under what condition HK is a subgroup of G?
   g) Lagrange theorem is applicable for which groups?
   h) Define center of a group. Is it a subgroup of the group G?
   i) How many subgroups of order 4 does D_4 have?
   j) Is the union of two subgroups of a group G also a subgroup of G?
   k) Define C(H) where H is a subgroup of the group G.
   l) Is every Abelian group cyclic?

Group-B

2. Answer any eight of the following questions within two or three sentences each. [2x8]
   a) Prove that the identity element of a group is unique.
   b) Show that U (10) is a group with the help of a table.
   c) In a group G, prove that $(a.\,b)^{-1}=b^{-1}.a^{-1}\ \forall a, b \in G$.

d) Prove that if G is an Abelian group under multiplication with identity e then H = {x^2|x = G} is a subgroup of G.

e) Define homomorphism of groups.

f) If ∅: G→$\bar{G}$ is a homomorphism, then prove that ∅ (e) = $\bar{e}$ where e and ē are the identity elements of G and $\bar{G}$ respectively.

g) Define order of an element of a group G. Hence find the highest order of an element in S_3.

h) If G is a group and H is a subgroup of G then prove that for a ∈ G the set aHa^{-1} = {aha^{-1}: h ∈ H) is a subgroup of G.

i) Define cyclic group. Hence prove that every cyclic group is Abelian.

j) If G is a group of finite order, then prove that $a^{0(G)} = e$, ∀a ∈ G.

GROUP-C

3. Write notes on any eight of the followings within 75 words: [3x8]

a) If in a group G every element is its own inverse, then prove that G must be Abelian.

b) If G is a group of finite order, then we can find a positive integer N such that a^N = e for a ∈ G. Prove it.

c) Prove that the intersection of two normal subgroups of a group G is also a normal subgroup of G.

d) If G is a finite group and His a subgroup of G, then prove that O(H)|O(G).

e) Define the kernel of an homomorphism. Hence prove that Kernel of a homomorphism is a normal subgroup of G.

f) With the help of a table, prove that U (5) is a group. What is the order of this group?

g) Find a noncyclic subgroup of A_8 that has order 4.

h) if Φ:G→$\bar{G}$ is a homomorphism, then prove that Φ (x^{-1}) = Φ $(x)^{-1}$ ∀x ∈ G.

i) If N is a normal subgroup of G, then prove that every left coset of N in G is again a right of N in G.

j) Show that a permutation with odd order must be an even permutation.

<u>GROUP- D</u>

4. Answer <u>any four</u> questions within 500 words each. [7x4]

a) If in a group G

$$(a.\ b)^2 = a^2.b^2 \quad \forall\ a, b \in G$$

then prove that G is abelian.

b) If G is a finite group and $a \in G$, then prove that $O(a)\ |O(G)$.

c) Prove that a group of order 3 must be cyclic.

d) Prove that

$$H = \left\{ \begin{vmatrix} 1 & n \\ 0 & 1 \end{vmatrix} |n \in Z \right\} \text{ is a cyclic subgroup of } GL(2, R).$$

e) Prove that every subgroup of an Abelian group is a normal subgroup.

f) If N and M are two normal subgroups of G, then NM is also a normal subgroup of G.

g) Prove that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

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