2020-21

Time - 3 hours
Full Marks – 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.

Group-A

1. Answer all questions or fill in blanks as required. [1x12]

   a) Give an example of a second order quasilinear partial differential equation.

   b) What is non-linear equation?

   c) What is wave equation?

   d) Define linear differential equation.

   e) P(x, y, z) p+ Q(x, y, z) q = R(x, y, z) is ________.

   (Choose the correct answer.)

   I) Lagrange's equation

   II) Charpit's equation

   III) Euler's equation

   IV) Pfaffian equation

   f) (λx , λy) = λ^n f(x, y) then f(x, y) is__________.

   g) A partial differential equation which is not quasi linear is said to be.

   (Choose the correct answer)

   I) Linear iii) Semi-linear

   II) Non-linear (iv) All of the above

   h) Obtain the differential equation if $\mathrm{Y} = \mathrm{A}\sin\mathrm{n} + \cos n$.

   i) Number of independent variables in partial differential equation are____________.

   j) What is Cauchy problem?

Group-B

2. Answer any eight of the following questions within two or three sentences each. [2x8]

   a) Define complete integral.

   b) Define a singular solution with a suitable example.

   c) Give an example of quasi-linear equation.

   d) Define D'Alembert's solution.

   e) What is non-homogeneous wave equation?

   f) Define homogeneous equation.

   g) Solution of the differential equation $\frac{dy}{dx}$= 2x subject to the conditions y(1) = 4. Prove it.

   h) What is the order of ODE

   $$\left(\frac{dy}{dx}\right)^3 = \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}?$$

   i) What is the general solution of the differential equation $(1 + x)dy - y\,dx = 0$.

   j) What is semi-linear partial differential equation?

GROUP-C

3. Write notes on any eight of the followings within 75 words: [3x8]

   a) Eliminate the function z from $z = e^{mn}f(x + y)$.

   b) Eliminate the arbitrary constant of $x^2 + y^2 + (z - c)^2 = a^2$.

   c) Find the general solution of the system by the trial solution method

   $$\left.\begin{aligned}\frac{dy}{dt} &= x + 3y\\ \frac{dy}{dt} &= 3x + 3y\end{aligned}\right\}$$

   d) Solve: $x\frac{\partial u}{\partial x} + y\frac{du}{dy} + z\frac{du}{dz} + xyz$.

   e) Solve the boundary value problem $\frac{d^2x}{dx^2}$+ 2y = 0, y(0) = 1, y(π) = 0.

   f) Find out the general solution of $1.u_x$=0.

   g) Define one-dimensional heat equation.

h) What is Laplace equation?

i) Solve: $y^2p - xyq = n(z - 2y)$.

j) Find the solution of the ordinary differential equation: $\frac{dy}{dx} = 2n + 1$

GROUP- D

4. Answer any four questions within 500 words each. [7x4]

a) Find the general solution of $zxp - zyq = y^2 - x^2$.

b) Use the separation of variables to solve the equation u_x^2+u_y^2=1.

c) Reduce the following equation into a Canonical form and find out the general solution $u_x - u_y = u$.

d) Solve the quasi-linear equation:

$$x(y^2 + u)\, u_x - y(x^2 + u)\, u_y = (x^2 - y^2)u.$$

e) Find the general solution of $(1 + x^2)u_x + u_y = 0$.

f) Solve the linear equation $yu_x + xu_y = u$.

with the Cauchy data $u(x, 0) = x^3$ and $u(y, 0) = y^3$.

g) Find out the general solution of Laplace equation in two-dimensions.

h) Find out the solution of initial boundary value problem

$$u_{tt} = u_{xx}, \quad 0 < x < \infty, t > 0$$

$$u(x, 0) = \cos\frac{\pi x}{2} \quad 0 \leq x < \infty:$$

$$u_t(x, 0) = 0, 0 \leq x < \infty.$$

$$u_t(x, 0) = 0 \quad t \geq 0.$$

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