2020-21

Time - 3 hours
Full Marks – 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.

Group-A

1. Answer all questions or fill in blanks as required. [1x12]

   a) Define Cartesian product of two sets.

   b) Define power set of set A.

   c) When two sets X and Y are said to be equivalent?

   d) Define tautology.

   e) Determine whether the following relation is a function with domain {1, 2, 3, 4}?

      f = {(1, 2), (2, 3), (4, 2)}

   f) What is well ordering principle of natural numbers.

   g) A zero matrix is always a square matrix. (True / False)

   h) To every linear transformation, there corresponds a unique matrix (True/False).

   i) Define involutory matrix.

   j) Span of x+ y=0 and x-y=0 in V_3 is V_3 (True/False).

   k) If U and W be two subspaces of a vector space V, then U ∪ W is always a subspace of V. (True/False)

   l) A subset of a linearly dependent set can never be linearly dependent. (True/False)

Group-B

2. Answer <u>any eight</u> of the following questions within two or three sentences each. [2x8]

a) Prove

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

b) Prove (A')' = A.

c) Prove that if square of a number is even then the number itself is even.

d) If f and g are functions R→R defined by f(x) = 2x-3, g(x)= $\frac{x}{x-1}$, then find fog.

e) Define ~ on Z by a~b iff 3a+b is a multiple of 4, prove that ~ satisfies symmetric properties.

f) In any vector space V, prove α0= 0 for every scalar α.

g) If U and W are subspaces of a vector space V, prove that U∩W is a subspace of V.

h) Let V be any vector space, then prove the set {v} is LD if v=0.

i) If α, β are scalars such that A = αB + βI, then prove that AB=BA.

j) Prove that $A-A^T$ is always a skew symmetric matrix.

GROUP-C

3. Write notes on any eight of the followings within 75 words: [3x8]

a) Simplify the statement [~(p v q)] [(~p) Λ q].

b) Prove that a ≡ b (mod 5) is an equivalence relation on Z.

c) Prove that composition of function is an associative operation.

d) Find the greatest common divisor of 630 and 196 by using Euclidean algorithm.

e) For sets A and B. prove that A ∩ B= A if and only if A⊂B.

f) Let S be a non-empty subset of a vector span V. Then prove the span of S is a subspace of V.

g) In an n-dimensional vector space V. prove any set of n linearly independent vectors is a basis.

h) Prove that if A is a non-singular square matrix, then A^1 is also non singular and $(A^T)^{-1} = (A^{-1})^T$.

i) Find kernel of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$$

j) If V has a basis of n elements, then every other basis for V also has n elements.

GROUP- D

4. Answer <u>any four</u> questions within 500 words each. [7x4]

a) Let f: X→Y. Then prove the relation $f^{-1}$is a function from Y to X if f is objective.

b) Show that

$$[(p \vee q) \vee ((q \vee (\sim r)) \wedge (p \vee r)] \Leftrightarrow \sim[(\sim p) \wedge (\sim q)]$$

c) If a ≡ x(mod n) and b≡y(mod n), then prove

I) a + b ≡ x+ y(mod n)

II) ab ≡ xy (mod n)

d) State and prove rank nullity theorem.

e) In a vector space V, if (V_1, V_2,……, V_n) generates V and if (w_1, w_2, w_3 ……w_n) is LI then prove $m \leq n$.

f) Invert the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

g) Determine the eigen values and the eigen vectors of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$$

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