2020-21

Time - 3 hours
Full Marks – 60

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.

Group-A

1. Answer all questions. [1x8]

   a) What is the value of $\Gamma(-1)$?

   b) Does Beta function satisfy symmetry property?

   c) Can derivative of an odd function be an even function?

   d) Find order and degree of differential equation

   $$x^2y'' + xy' + by = 0.$$

   e) Define a periodic function.

   f) What is systematic error?

   g) What is the value of odd function at the origin?

   h) Define exact equation.

Group-B

2. Answer any Eight of the following questions within two or three sentences each. [$1\frac{1}{2}$x8]

   a) State Dirichlet's conditions.

   b) Show that derivative of an odd function is an even function.

   c) Locate the singular point of differential equation $x^2y'' + (2-x)\,y' = 0$.

   d) Determine the nature of point x=0 for the equation $y'' + (\sin x)\,y = 0$.

   e) State the relation between Beta function and Gamma function.

   f) Compute: $\dfrac{\Gamma(3)\Gamma(4)}{\Gamma(5)}$

g) Find $\Gamma(\frac{3}{2})$

h) Using generating function for Legendre polynomial, show that

$P_n(1) = 1$.

i) State the orthogonality property of Hermite polynomial.

j) Show that $\Gamma(0)=\infty$.

GROUP-C

3. Write notes on any eight of the followings within 75 words: [2x8]

a) Write a note on Fourier sine series.

b) Mention the usefulness of Fourier series.

c) State the convergence property of Fourier series.

d) Find the value of $\Gamma(\frac{1}{4})\,\Gamma(\frac{3}{4})$.

e) Show that $\beta(m+n) = \frac{m+n}{m}\,\beta\,(m, n + 1)$.

f) Define error function of x. Show that error function of infinity is one.

g) Show that $P(-x) = (-1)^n P_n(x)$.

h) Prove that $H_2(x) = 4x^2-2$.

i) Evaluate $\int_0^\infty e^{-x^2} x^3.dx$.

j) Express x^2 in terms of Legendre's polynomial.

GROUP- D

4. Answer <u>any four</u> questions within 500 words each. [6x4]

a) Find the Fourier series on the interval $(0, 2\pi)$ for the function $f(x) = x$.

b) Derive Rodrigue's formula for Legendre's polynomial.

c) Write notes on (i) Cosine integral, (ii) Sine integral, (iii) Exponential form of Fourier series. [2 x 3]

d) Solve the Hermite's differential equation by power series method.

e) State and prove the orthogonality property of Hermite polynomial.

f) Show that ß (m, n + 1) = $\frac{n}{m+n}$ ß (m, n).

g) Find $\int_0^1 (x \log x)^3\, dx$.

h) Find the solution of Laplace equation in cylindrical coordinates by using the method of separation of variable.

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KACK - 2021